August 13, 2024

Use of Binomial Statistics for Assessment

Brown AND Caldwell

Clifton F. Bell

757.518.2456

cbell@brwncald.com

Underlying goal for this topic: Quantify and control type 1 and type 2 assessment errors

Water Body is Assessed as	Water Body is Truly Attaining	Water Body is Truly Impaired
Impaired	Type I error	Correct decision
Attaining	Correct decision	Type II error

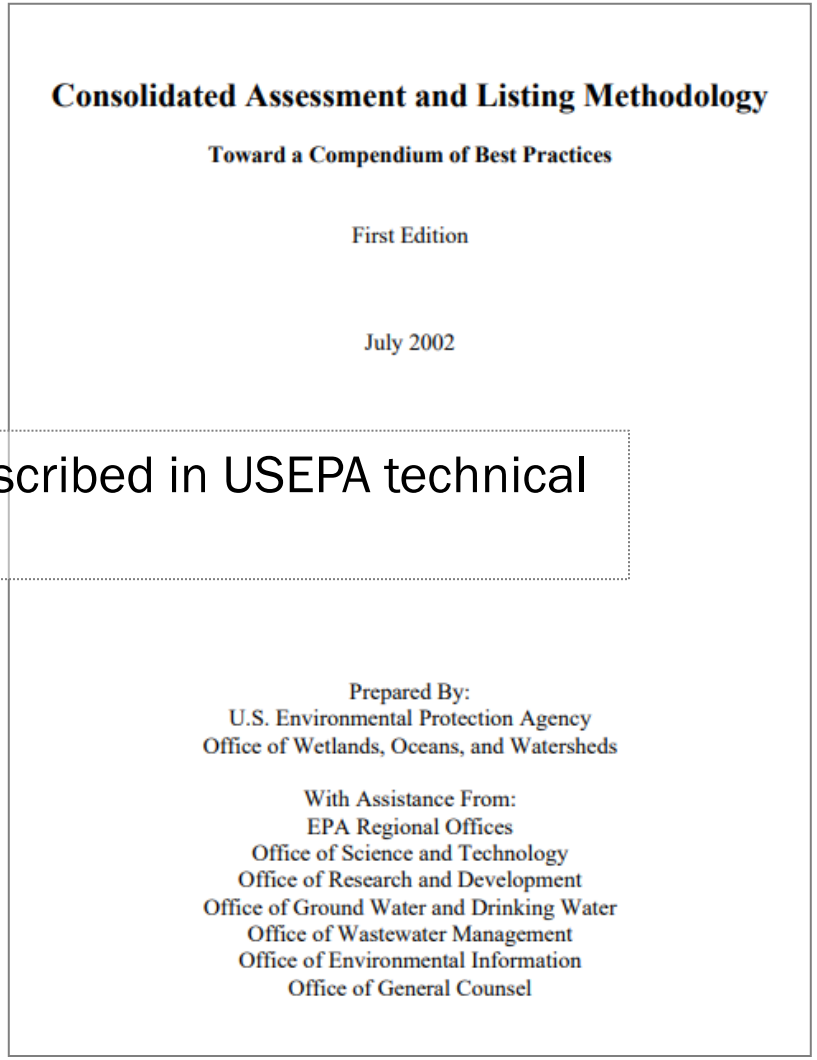
Excerpt from EPA (2006) guidance on integrated reporting

- EPA recommends that, when picking the decision rules and statistical methods …states …minimize the chances of …[the] two following errors:
  - Concluding the segment is impaired, when in fact it is not, and
  - Deciding not to declare a segment impaired, when it is in fact impaired.
- States should specify in their methodology what significance level they have chosen to use

A number of states use the binomial for water quality assessment (for at least some criteria)

- California
- Florida
- Kansas
- Oregon
- Montana
- Mississippi
- New Jersey
- North Carolina
- Texas
- *Others*

…and also described in USEPA technical documents



Consolidated Assessment and Listing Methodology

Toward a Compendium of Best Practices

First Edition

July 2002

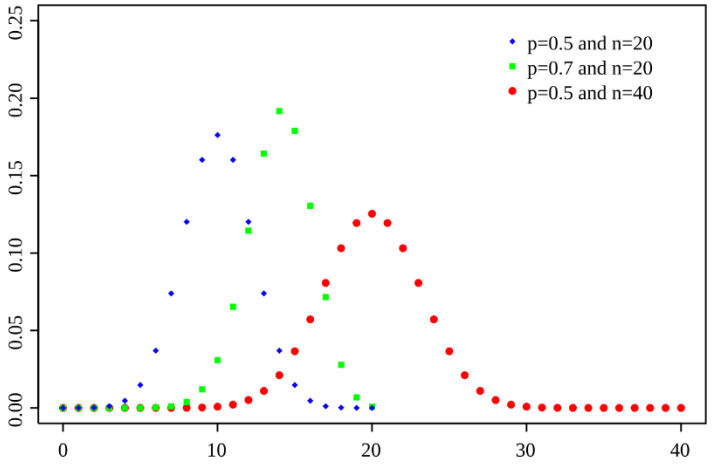
Prepared By:
U.S. Environmental Protection Agency
Office of Wetlands, Oceans, and Watersheds

With Assistance From:
EPA Regional Offices
Office of Science and Technology
Office of Research and Development
Office of Ground Water and Drinking Water
Office of Wastewater Management
Office of Environmental Information
Office of General Counsel

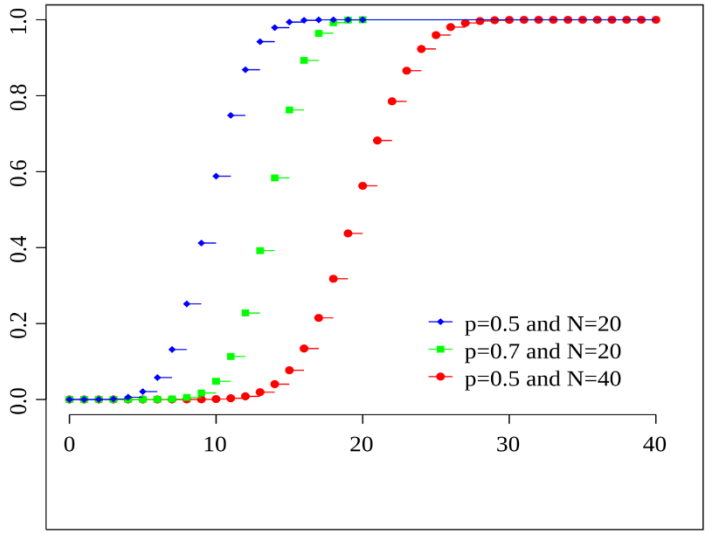
The Binomial Distribution

- Discrete probability distribution of number of “successes” in a sequence of n independent trials
- Each trial has two possible outcomes
- Probability of success = p
- Probability of failure = q = 1- p

Probability mass function



Cumulative distribution function



The Binomial Distribution Formula

$$P(x) = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

Where:

n = number of trials

x = number of successes of interest

p = probability of getting a success in one trial

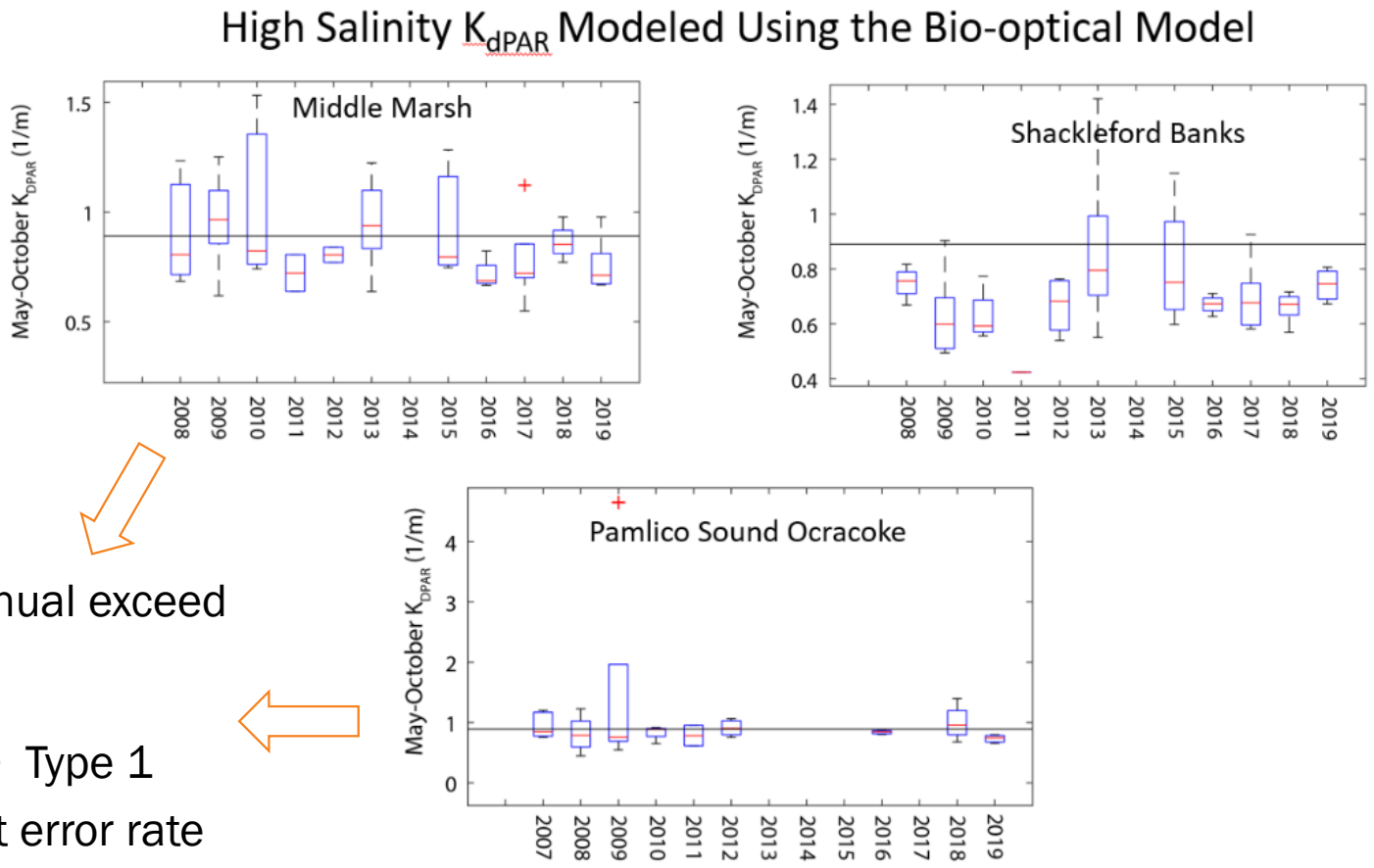
q = 1 – p = the probability of getting a failure in one trial

Example: Justifying a 1-in-3 allowable exceedance rate for marine chlorophyll-a criteria (Florida DEP, 2012)

- Criteria derived from healthy reference condition, so low concern over Type II errors
- DEP desired to limit Type I error probability to 10%
- The reference condition itself had a 20% probability of exceeding the criteria (set at 80th percentile)
- Binomial stats: A 1-in-3 year allowable exceedance equates to a ~10% Type I error probability

X (# exceedances)	P(X) (probability of X exceedances in 3 trials)	Cumulative P (probability of ≤X exceedances)
0	0.512	0.512
1	0.384	0.896
2	0.096	0.992
3	0.008	1.000

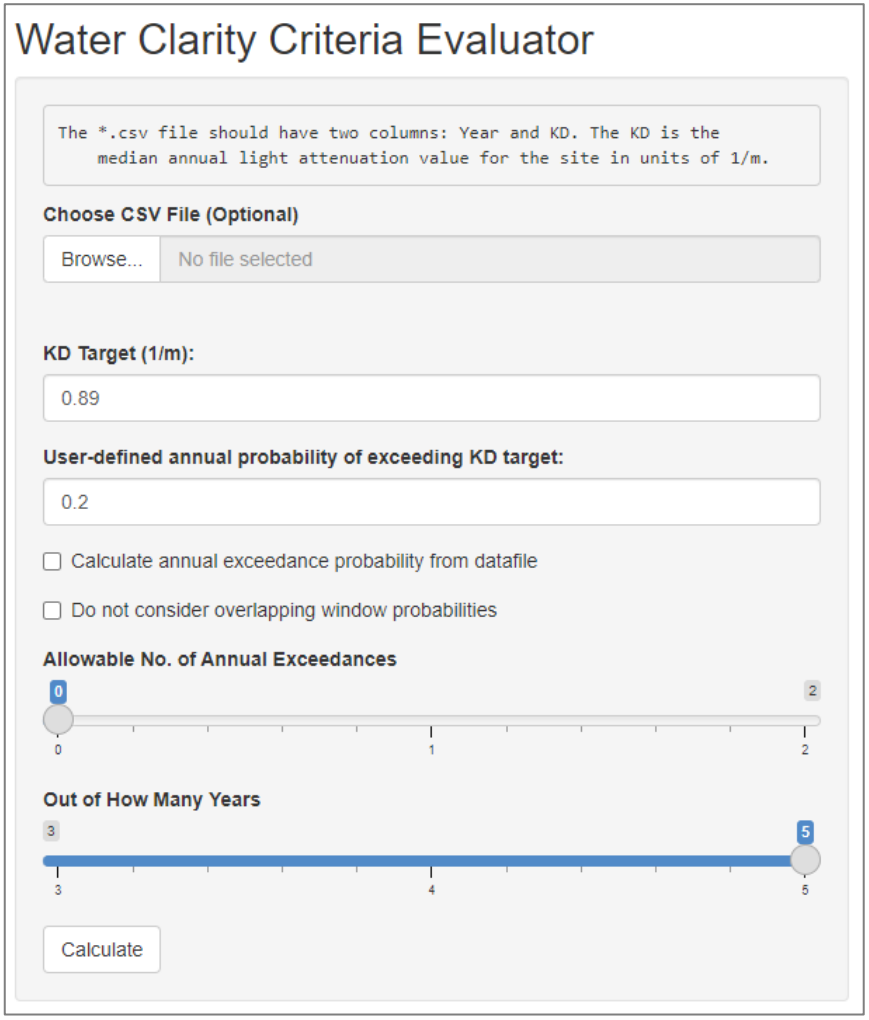
Example: Reference SAV Sites had a 20% annual probability of failing clarity criteria



- ~20% annual exceed rate
- ~67% Type 1 assessment error rate (5-yr assessment period)

Source of charts: Hall (2022b)

Application allows exploration of Type I error rates based on K_d target, frequency component



Annual probability of exceeding KD Target: 20 %

Probability of exceeding the target more than the allowable frequency over the multi-year assessment period: 67 %

Example: NC's use of the binomial stats for water quality assessment

- Information here taken from *NC 2024 303(d) Listing and Delisting Methodology*
- https://edocs.deq.nc.gov/WaterResources/DocView.aspx?dbid=0&id=3075212&cr=1

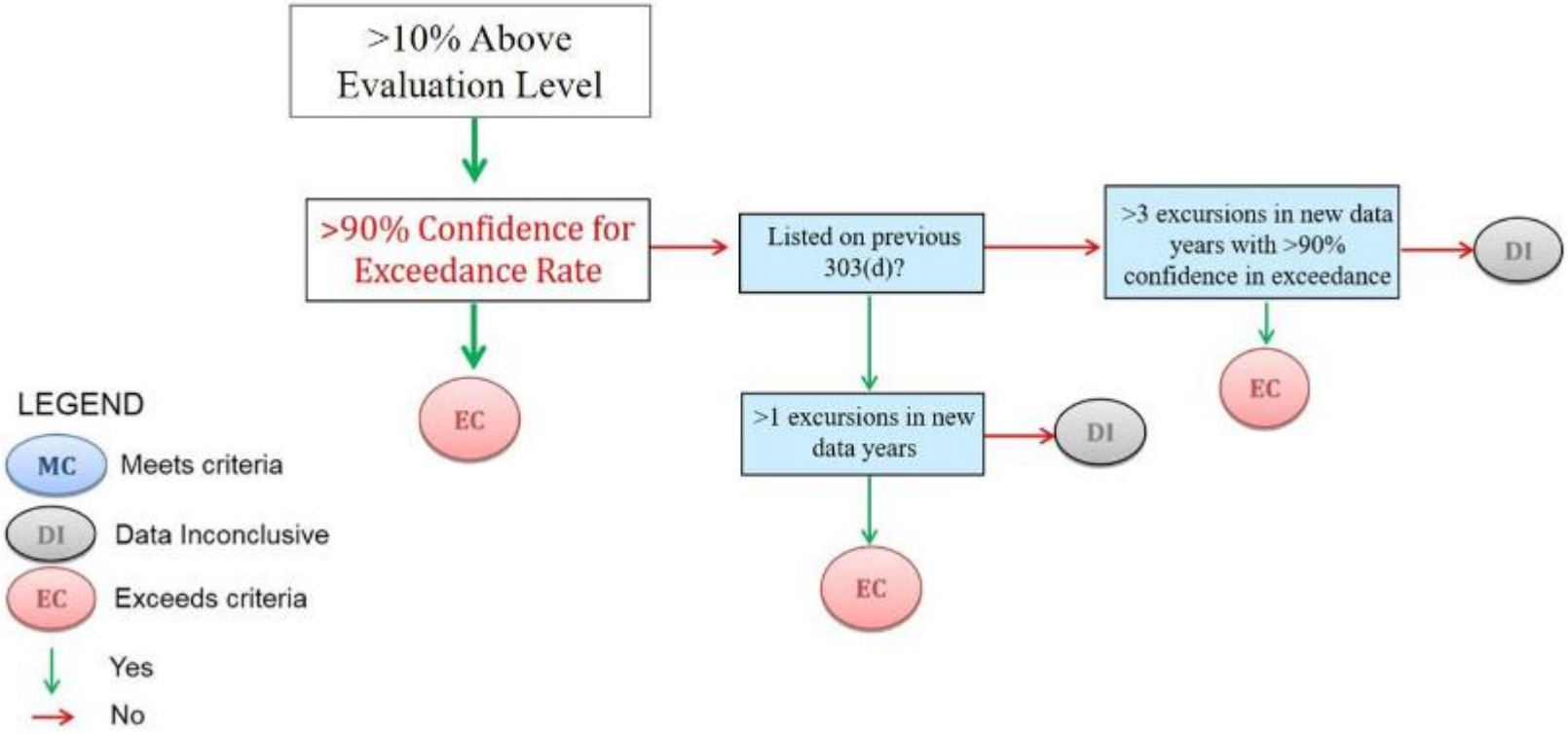
NC Listing Basis

- “NC will use a nonparametric hypothesis testing approach based on the binomial distribution.”
- “The binomial method allows a quantifiable level of statistical confidence (90%) for listing decisions, which provides a 10% probability of listing an assessment unit when it should not be listed.”
- “The null hypothesis is that the overall exceedance probability is less than or equal to the 10% exceedance allowance.”

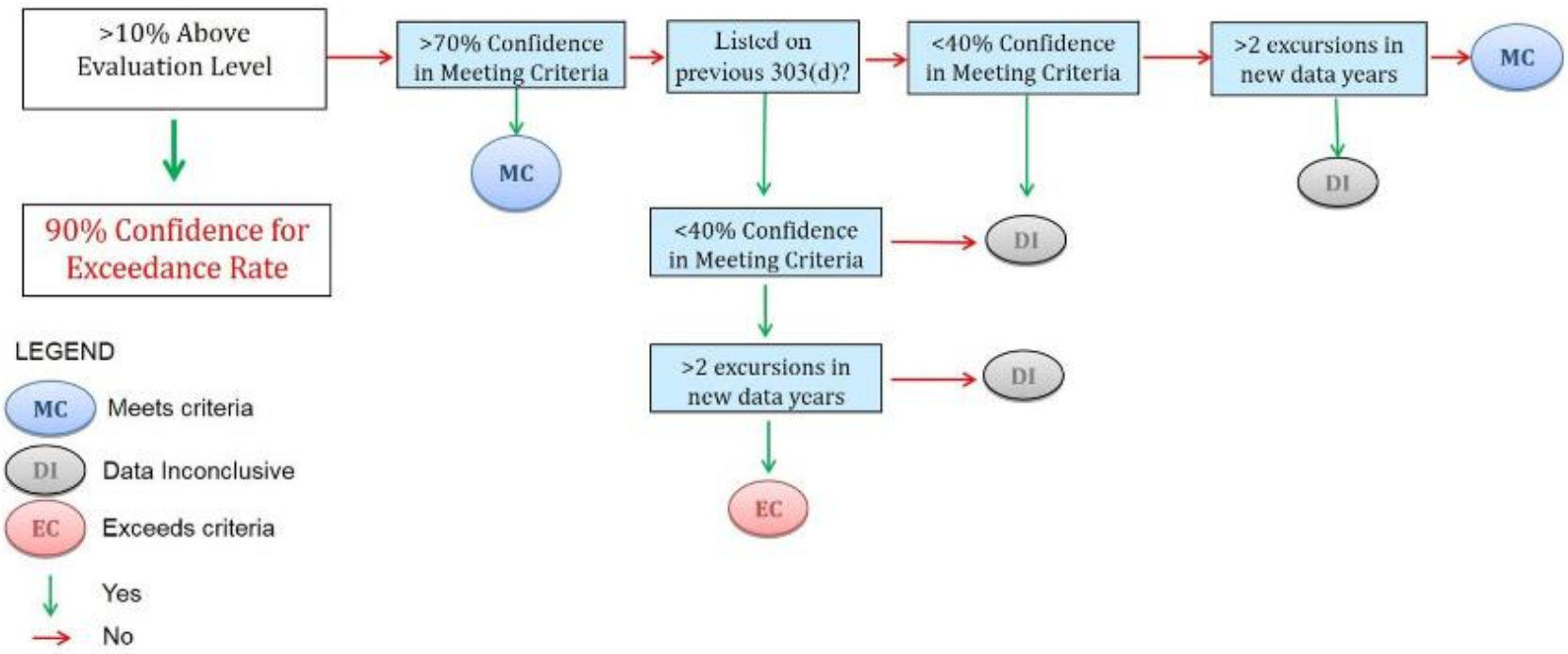
Listing/De-listing Decisions Based on:

- Level of confidence that null hypothesis (of ≤ 10% exceedances) should be rejected
  - To list: 90% confidence of >10% exceedances
  - To de-list: 40-70% confidence of ≤ 10% exceedances
- Listing status in previous integrated report
- Number of excursions in new data (most recent two years included)

Flow Chart for >10% Exceedance



Flow Chart for ≤10% Exceedance



Example 1

- Current listed as not impaired
- 5 of 30 samples exceed = ~16.7%
- Binomial says if “true” exceedance rate is 10%, only ~8% probability of getting 5 or more exceedances out of 30
- Reject null hypothesis, list water body

Example 2

- Site currently listed as impaired
- 20 samples in assessment period
- 3 exceed criterion = 15% of samples
- Binomial says if “true” exceedance rate is 10%, only ~32% probability of getting 3 or more exceedances out of 20
- But >1 excursion in new data
- Keep impairment listing

Examples of related methods

- Hypergeometric test
  - Washington Dept. of Ecology (https://apps.ecology.wa.gov/publications/documents/1810035.pdf)
  - Analogous to binomial but without replacement
  - Value of interest is “days in a year with an exceedance” instead of % exceedances
- Sequential probability ratio test
  - Proposed by Chen and others (2017)
  - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5331907/
  - Authors claim fewer samples required than binomial

Summary of concepts embedded in these precedents (beyond the specific method)

- Hypothesis testing of probability of impairment
- Quantifying:
  - …the % exceedance associated with listing/not listing
  - …the % confidence required
- Existing listing status affects decisions: Confidence needed to change status
- Asymmetry in the required confidence for listing/de-listing: Higher confidence needed to commit TMDL resources (?)
- Quantitative basis for concluding "data inconclusive" ➔ category 3